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( Final Report)  
Theory of the Cylindrical Dipole on a Sphere

L. L. Tsai

D. V. Otto

Prepared by  
The Ohio State University

**ElectroScience Laboratory**

(formerly Antenna Laboratory)  
Department of Electrical Engineering  
Columbus, Ohio 43212

for

Massachusetts Institute of Technology  
Lincoln Laboratory

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## ABSTRACT

A logical and straightforward approach to the problem of the cylindrical dipole mounted symmetrically on a conducting metallic sphere is given. The method used replaces the conducting sphere by a system of properly chosen interior sources whose coefficients are determined by Fourier Transform theory, an aperture condition, and variationally optimized point-matching. The results show agreement with experiment within about 5%. Extension of the method to other shapes is considered briefly.

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Franklin C. Hudson  
Chief, Lincoln Laboratory Office

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Final Report  
THEORY OF THE CYLINDRICAL DIPOLE ON A SPHERE

I. INTRODUCTION

A structure which is useful as an active self contained field probe, scatterer or transmitter is that of the dipole whose central region is a coaxial sphere, cylinder or other symmetrical shape. Typical geometries are shown in Fig. 1. To date there has been no accurate solution of this problem. An approximate solution for the case of scattering by loaded wires attached to a sphere was given recently by Vincent and Chen,<sup>1</sup> but their solution approximated the interaction between sphere and wires by imaging in flat conducting ground planes. This is a reasonable assumption only when the length of the wire attached to the sphere is much less than the sphere diameter. Apart from the other approximations inherent in their method, therefore, it is not surprising that agreement between theory and experiment is more qualitative than quantitative.

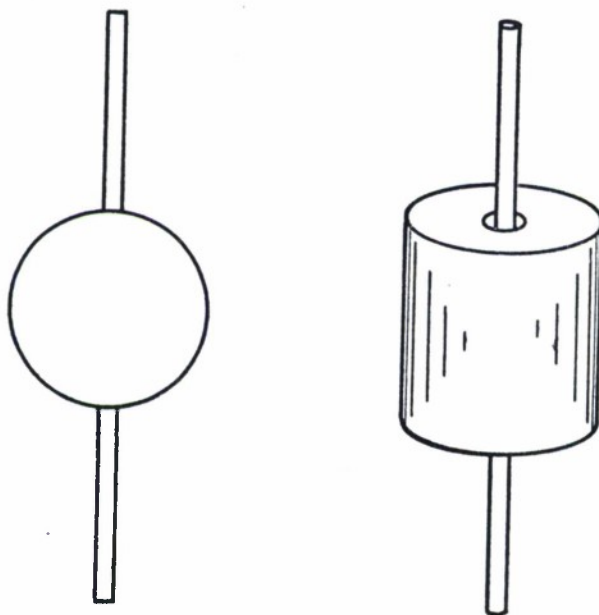


Fig. 1. Typical modified dipole geometry.

In the present study, a fairly simple technique is developed for the analysis of the problem of the modified dipole considered as a transmitter. The method is based on (a), the Fourier Transform treatment of the cylindrical antenna problem with realistic source,<sup>2,3</sup> (b), the static image system for a sphere, and (c), point-matching. The idea is that the

sphere-cylinder system can be replaced by an equivalent system (as far as external fields are concerned) consisting of a simple perfectly conducting rod with a source system of (for the sake of convenience) magnetic current frills as shown in Fig. 2d. With this system the interior sources are adjusted so that  $\underline{n} \times \underline{E}$  is approximately zero on the spherical surface. Then the external fields approximate the real situation, while the internal fields are a mathematical convenience.

Experimental results are given for the case of a dipole on a sphere and also on a cylinder, showing in the first case that the theory gives accurate results, and in the second that if the shape of the sphere is changed, the resultant modification to the admittance (and therefore the far field) is small. Theoretical analysis of the dipole on a coaxial cylinder using a different equivalent source system is also briefly discussed.

## II. THEORY

The problem to be analyzed is shown in Fig. 2a; a hemisphere of radius  $a_s$  is mounted on a ground plane. A cylindrical wire, radius  $a$ , length  $h_c$ , is fitted coaxially as shown with a coaxial aperture of radius  $b$ . Figure 2b shows the static image of the wire in the sphere along with the equivalent magnetic current source assuming static image concepts apply because the sphere is electrically small ( $a_s < \lambda/16$ ). Evidently the image of the cylinder is a surface whose generator is an arc of a circle of radius  $a_s^2/2a$  with center lying in the equatorial plane and distant  $a_s^2/2a$  from the antenna axis. On this surface, tangential  $E$  is zero. Analytically, however, it is more convenient to deal with a cylindrical surface as the image source. This cylinder can be taken to be the projection of the wire surface through the sphere (i.e., radius  $a$ ) as shown in Fig. 2c. Now, however, it is necessary to have some knowledge of the relationship between the equivalent currents on the cylinder and the "real image". If  $a \ll a_s$ , then there is little difference between the cylindrical surface and the curved image surface. Thus over most of the range  $|z| < a_s$ , it is reasonable to assume that on the cylinder tangential  $E$  is zero. Near  $z = \pm a_s^2/h$  (i.e., adjacent to the images of the ends of the wires), tangential  $E$  on  $r = a$  is manifestly non zero. Some idea of the form  $E_z$  in this region can be gained from the following analysis: The line A-A in Fig. 2b is the static image locus of points on the circle B-B (radius  $a_s^2/2a$ ) outside the sphere. Assume the field near the end of the antenna is given approximately by a sinusoidal line distribution of current along the antenna axis. Then the  $E$ -field near the end of the wire is radial with origin at the end, and is proportional to  $1/x$  with  $x$  the distance from an end, as shown in Fig. 2c, ( $x \ll \lambda$ ). It is clear, therefore, that  $E_t$  will have maxima on either side of C ( $\underline{t}$  and C shown in Fig. 2c). The result is that on the image cylinder one can expect  $E_t$  to have maxima at

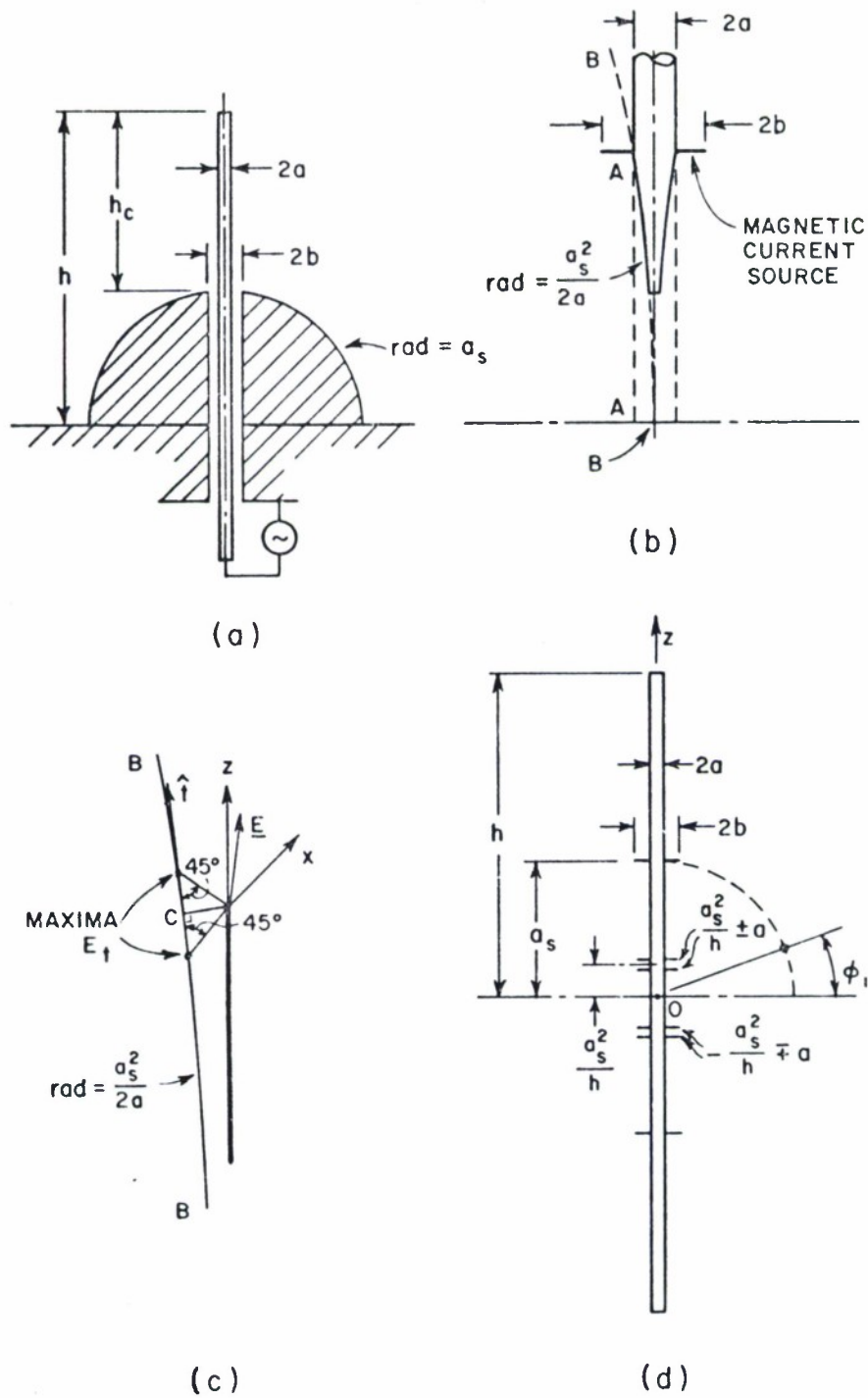


Fig. 2. (a) Experimental set-up  
 (b) Equivalent (static) image system  
 (c) Image considerations, and  
 (d) Final equivalent system .

$$(1) \quad z = \pm \left( \frac{a_s^2}{h} \pm a \right) .$$

It is appropriate therefore to place interior (frill) sources as shown in Fig. 2d at these locations.

It remains only to determine the amplitude of these sources. First, we assume an aperture condition

$$(2) \quad I'(\pm a_s + \Delta) = I'(\pm a_s - \Delta) .$$

This is equivalent to the assertion that the field form near the aperture is the same as if the wire were mounted on a flat ground plane. Second, at one point (or latitude in fact) on the sphere, tangential E, (i.e.,  $E_\phi$ ) is made zero, so that

$$(3) \quad E_\phi(\phi_1) = 0$$

with  $\phi_1$  determined variationally as shown in the Appendix. Thus we have the situation shown in Fig. 2d - a simple, perfectly conducting rod with three pairs of magnetic current sources, one pair of which ( $z = \pm a_s$ ) are of known strength, with the coefficients for the remainder being determined from Eqs. (2) and (3).

A zero order (5% accuracy) approximation for the current is given from Fourier transform theory<sup>3</sup> by,

$$(4) \quad \begin{aligned} I(z) = & I_f(z + a_s) + I_f(z - a_s) + C_1 \{I_f(z + a_s^2/h + d) + I_f(z - a_s^2/h - d)\} \\ & + C_2 \{I_f(z + a_s^2/h - d) + I_f(z - a_s^2/h + d)\} \\ & + C_3 \{I_R(z + h) + I_R(z - h)\} , \end{aligned}$$

where  $d = a$  and <sup>2</sup>

$$(5) \quad I_f(z) = \frac{2\pi i}{Z_0 \ln(b/a)} \int_0^\infty \frac{e^{-kz\sqrt{v^2-1}}}{v\sqrt{v^2-1}} \frac{\{J_0(vka)Y_0(vkb) - J_0(vkb)Y_0(vka)\}}{\{J_0^2(vka) + Y_0^2(vka)\}} dv$$

and<sup>3</sup>

$$(6) \quad I_R(z) = k \int_0^\infty \psi(z') e(z') I_e(z - z') dz' .$$

In Eq. (6),  $\psi(z)$  is a function which takes into account the edge effect at the ends of the antenna;

$$(7) \quad \psi(z) = \{1 + (z/a)^2\}^{1/6} (z/a)^{-1/3} ,$$

and  $e(z)$  is the field on the axis due to a piecewise-sinusoidal current on the antenna along with a square law ( $r^2$ ) current on the flat end;

$$(8) \quad e(z) = \left[ (1 - kz) \frac{e^{-ik\sqrt{z^2 + a^2}}}{k\sqrt{z^2 + a^2}} + e^{-ikz} \right] .$$

Also,  $I_e(z)$  is the exterior Green's function for an infinite cylinder,<sup>3, 4</sup> and in order to determine the third constant in Eq. (4), we have the end condition that

$$(9) \quad \begin{aligned} 0 = & I_f(h + a_s) + I_f(h - a_s) + C_1 \{I_f(h + a_s^2/h + d) \\ & + I_f(h - a_s^2/h - d)\} + C_2 \{I_f(h + a_s^2/h - d) \\ & + I_f(h - a_s^2/h + d)\} + C_3 \{I_R(0) + I_R(2h) + C_e\} \end{aligned}$$

where

$$(10) \quad C_e = 2 \int_0^\infty \psi(z) e(z) I_i(z) dz$$

and  $I_i(z)$  is the interior Green's function for a perfectly conducting hollow cylinder.<sup>3</sup>

Finally it is necessary to evaluate the electric field at the surface of the sphere. The most convenient technique to accomplish this is to use a piecewise-sinusoidal representation for the current.<sup>5</sup> Assuming that  $a/r_s \ll 1$ , (where  $r_s$  is the distance from the antenna axis to a point on

the spherical surface), so that the current can be considered to be a line source, we have

$$(11) \quad E_z(r, z) = 30i \sum_{j=1}^N \Delta I_j' \frac{e^{-ikR_j}}{kR_j} ,$$

$$(12) \quad E_r(r, z) = - \frac{30i}{r} \sum_{j=1}^N \Delta I_j' \frac{e^{-ikR_j}}{kR_j} (z - z_j) ,$$

where

$$(13) \quad \Delta I_j' = \lim_{\epsilon \rightarrow 0} [I'(z_j - \epsilon) - I'(z_j + \epsilon)] .$$

Also,  $I'(z_1 - \epsilon) = I'(z_N + \epsilon) = 0$ , and the condition,  $I(h) = I(-h) = 0$ , is imposed with only small resultant error.

There is a contribution to the electric field from the magnetic currents as well as from the electric currents. This can be determined from the equation;

$$(14) \quad \underline{E} = - \frac{1}{\epsilon_0} \nabla \times \underline{M}$$

where

$$(15) \quad \underline{M} = \frac{\epsilon_0 \hat{\theta}}{2\pi} \int_a^b \int_0^\pi L_\theta(r) \frac{e^{-ikR}}{R} \cos \theta \, r \, dr \, d\theta ,$$

and

$$(16) \quad L_\theta(r) = - \frac{1}{r \ln(b/a)} , \quad a < r < b .$$

Equation (15) is readily evaluated numerically, and  $\underline{E}$  can be obtained via Eq. (14) by simple numerical differentiation.

### III. RESULTS

The theory just outlined was programmed for evaluation on a 7094 computer. The results for the admittance are shown in Fig. 3 compared with some experimental values. The experimental results were obtained by means of an impedance bridge connected via a carefully designed coaxial feed to the antenna which was mounted horizontally on a fairly small square ground plane as shown in Fig. 4. The frequency was nominally 375 MHz but was assessed at  $370.1 \pm 0.4$  MHz by comparing the theoretical and experimental maxima, minima and zeros for the case of the simple cylindrical monopole antenna with a precise theory<sup>6</sup> which takes into account contributions from finite conductivity, finite size of the ground plane, and reflections from the ground. The admittance in this case is

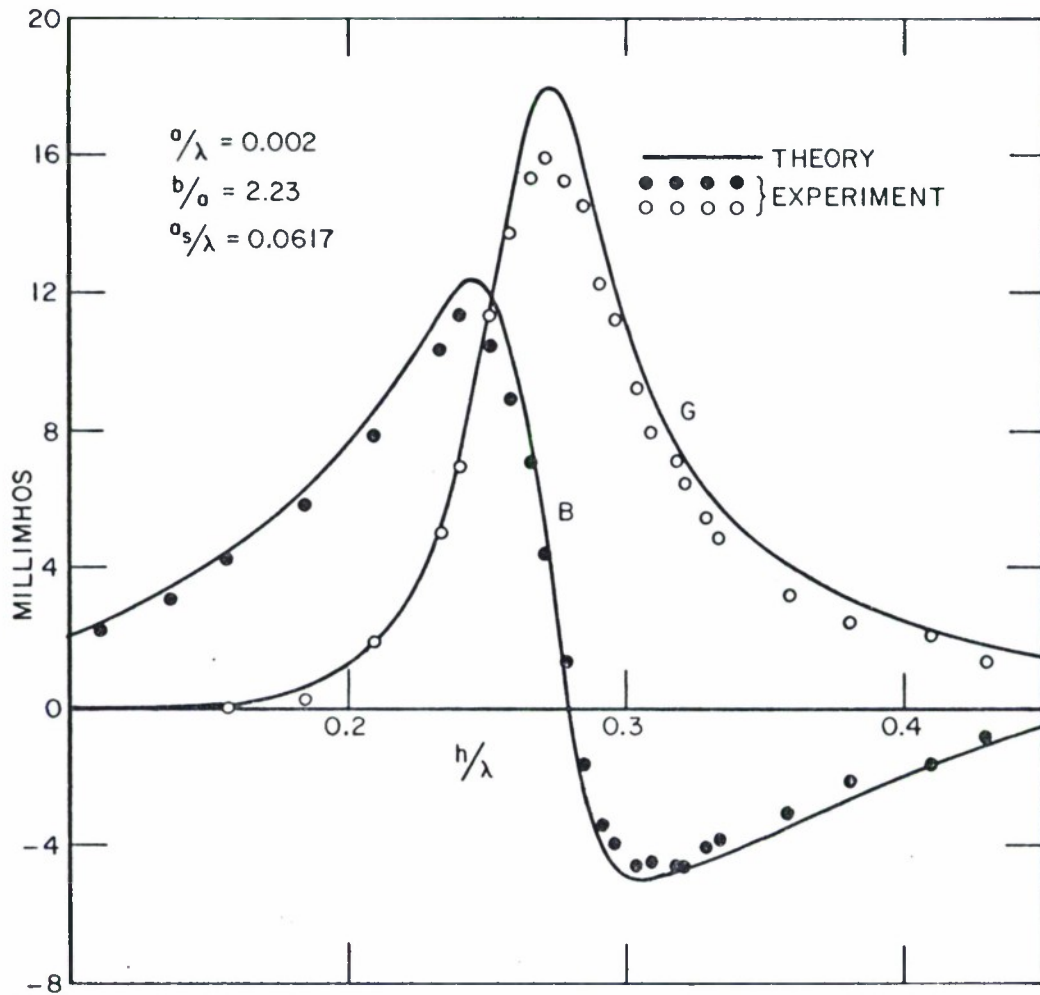


Fig. 3. Theoretical and experimental results for the dipole-sphere admittance.

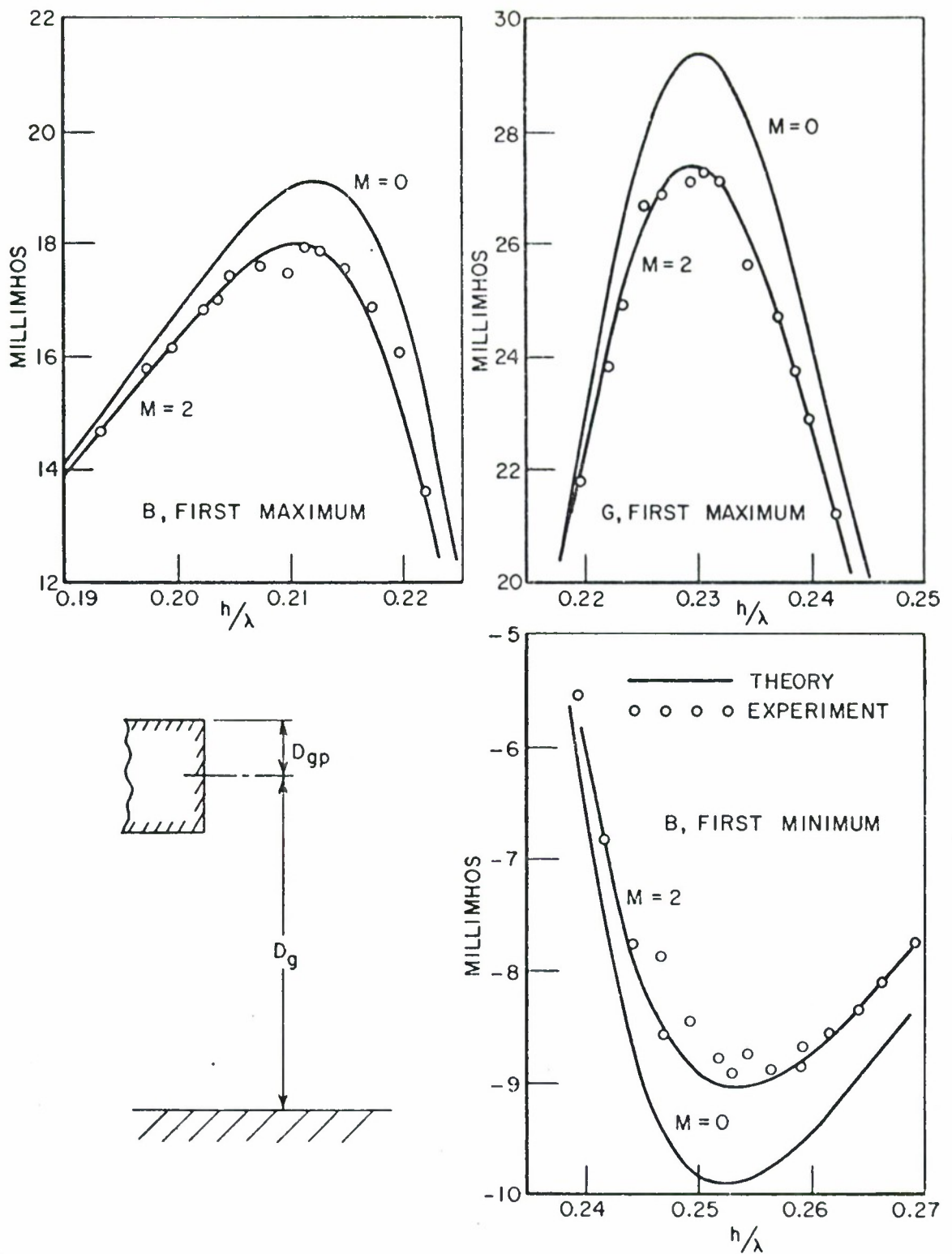


Fig. 4. Theoretical and experimental results for the admittance of the cylindrical antenna of Fig. 3 mounted with similar feed without the hemisphere on the ground plane. (Side length of square ground plane,  $2 D_{gp} = 2.5\lambda$ ; height of antenna above ground,  $D_g = 6.25\lambda$ .)

shown for the purpose of comparison in Fig. 4. The case  $M=2$  corresponds to the precise theory mentioned, whereas  $M=0$  corresponds to the zero order Fourier transform theory used in the present analysis. It is to be noted, particularly, that the discrepancy between the cases  $M=0$  and  $M=2$  is similar to the difference between the theoretical and experimental results shown in Fig. 3. We conclude, therefore, that the results are valid and, indeed, very satisfactory from an engineering point of view.

As mentioned in the previous section, the location of the point matching node was accomplished numerically by the method outlined in the Appendix. Interestingly enough,  $\phi_1$  was such that the nodal latitudes were non critically located on planes passing through the images of the ends of the antenna. The location of the interior frill sources was also checked in a similar manner. It was found in general that if the frills were located at

$$(17) \quad z = \pm (g \pm d) \quad ,$$

as defined in Eq. (1), then  $d$  was not critical but  $g$  should definitely be equal to  $a_s^2/h$ .

Figure 5 shows the tangential and normal components of  $\underline{E}$  on the surface of the sphere. It is a useful check on the results to note that  $E_R/E_\phi \gg 1$  so that the field is approximately normal to the surface as required.

The coefficients  $C_1$  and  $C_2$  tend to be equal and opposite and of an amplitude which roughly seems to follow  $Y$ . This amplitude is inversely proportional to  $d$  in Eq. (17) and, at  $h/\lambda = 0.275$ , is of magnitude about 41V when  $d = a$ , and about 11V when  $d = 3.5a$ . The corresponding values of the admittance in the two cases are,  $17.92 + i 2.32$  and  $18.30 + i 2.76$  mmho.

In Fig. 6, experimental results are shown for a dipole mounted coaxially on a cylinder. Also shown are the experimental results for a dipole on a sphere of the same radius as the cylinder. The change in admittance due to the change in shape is quite small. The admittance peaks become slightly smaller and the resonant length of the system is slightly larger, thus indicating that modified dipole characteristics are not highly sensitive to the shape of the central conducting body.

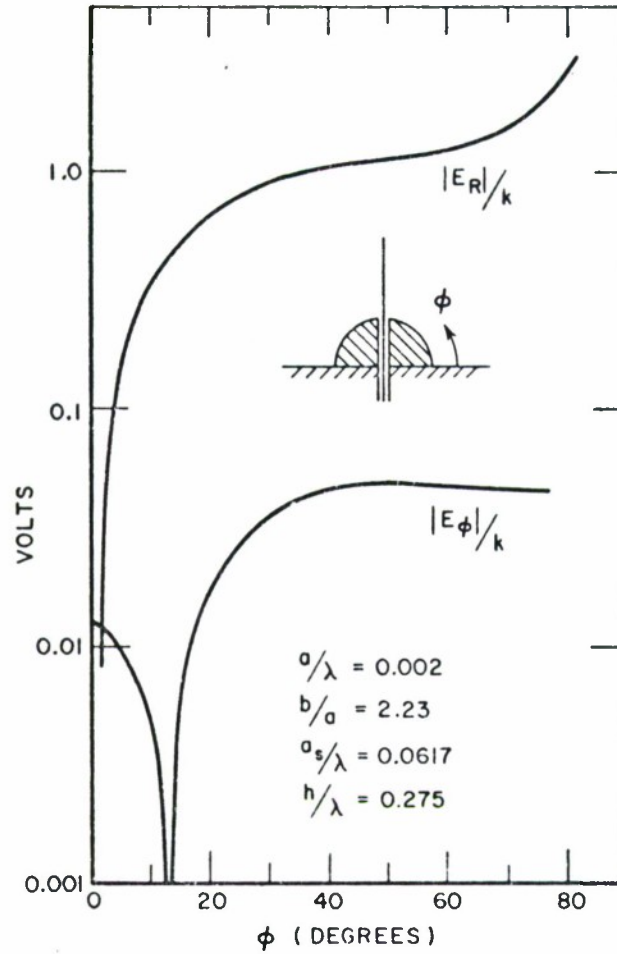


Fig. 5. Normal ( $E_R$ ) and tangential ( $E_\phi$ ) components of the theoretical electric field on the surface of the sphere.

#### IV. EXTENSION TO DIPOLE ON A CYLINDER

A logical extension would be the analysis of the dipole mounted on a coaxial cylinder using an equivalent magnetic frill source distribution similar to that employed for the sphere. For this problem, additional magnetic ring currents are located near the corners (as shown in Fig. 7) to satisfy the boundary conditions in that region. Preliminary data indicate that a satisfactory admittance calculation for a dipole on a cylinder can be obtained in this manner. For this problem, however, the method

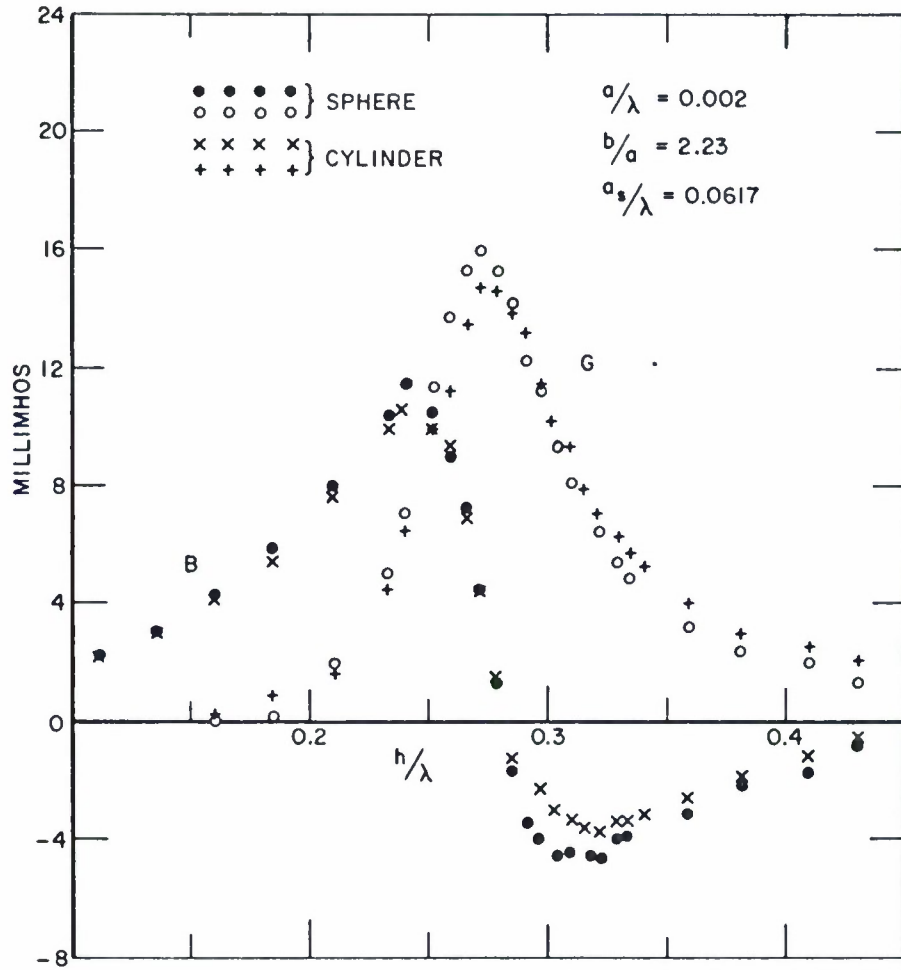


Fig. 6. Experimental results for a dipole on a cylinder of radius  $a_s$ , length  $2a_s$  compared with the experimental results for the dipole on a sphere of radius  $a_s$  (Fig. 3).

of static imaging does not readily yield the optimum locations for the frill sources. Consequently a different method has been developed to optimize the solution. The tangential fields, integrated over the conducting surface of the cylindrical modification, provides a figure of merit for the accuracy of our solution. By minimizing the quantity

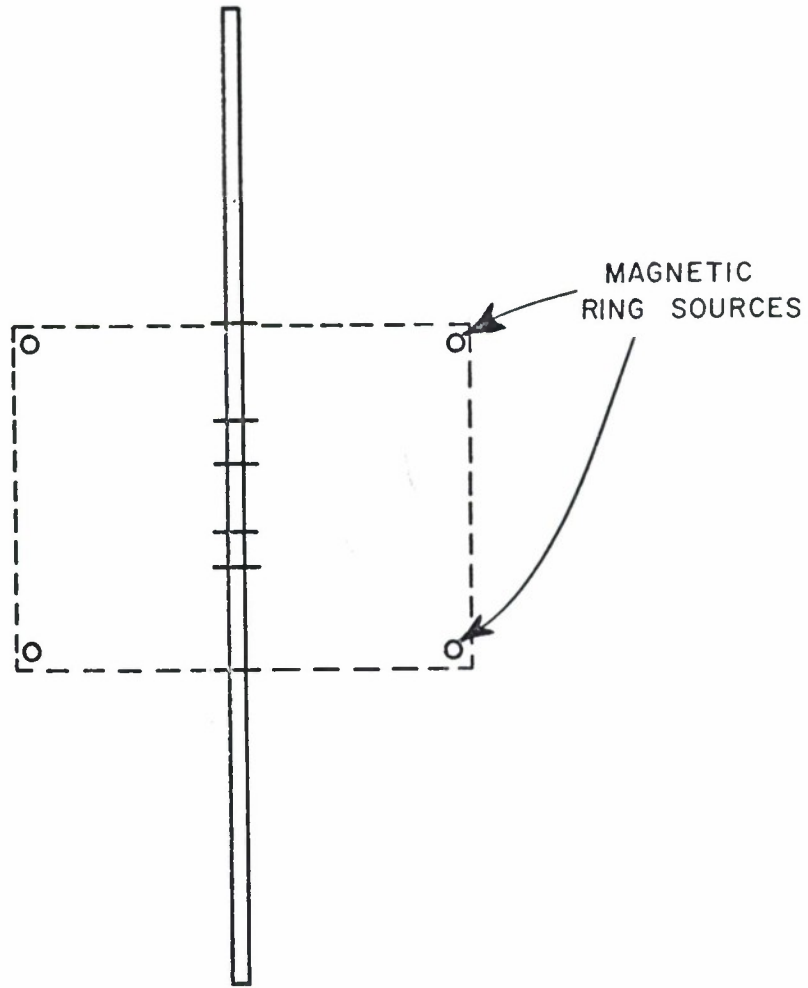


Fig. 7. Dipole antenna mounted on a coaxial cylinder, illustrating the equivalent internal sources.

$$(18) \quad Q = -\frac{1}{I^2} \iint \underline{J} \cdot \underline{E} \, ds,$$

where  $\underline{E}$  and  $\underline{J}$  are the electric field and currents on the cylindrical surface, a sensitive method for optimizing the locations of the frill and ring sources as well as the matching points can be achieved. This figure of merit concept has been applied to the modified dipole on a sphere and the results substantiated the findings by the method of static imaging used for that case.

## V. CONCLUSIONS

A logical and straightforward approach to the problem of dipole mounted symmetrically on a metallic sphere has been given. The results compare extremely well with experiment, leading one to the conclusion that problems of antennas mounted on three dimensional conducting bodies may be solved by replacing the conducting body by a system of suitably chosen interior sources.

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## APPENDIX

A simple condition on the tangential field on the sphere is that

$$(19) \quad R = \int_{-\pi+\beta}^{\pi-\beta} E_{\phi} d\phi$$

should be minimized assuming  $E_{\phi}$  does not oscillate violently). Since the current on the antenna, Eq. (4), can be written

$$(20) \quad I = I_0 + D_1 I_1 + D_2 I_2$$

where  $D_1, D_2$  are determined by Eqs. (2) and (3), and are functions of  $\phi_1$  (and any other parameters involved), we have

$$(21) \quad R = \int_{-\pi+\beta}^{\pi-\beta} \{ E_{\phi 0} + D_1 E_{\phi 1} + D_2 E_{\phi 2} \} d\phi .$$

In order that  $R$  should be a minimum

$$(22) \quad 0 = \frac{\partial D_1}{\partial \phi_1} \int_{-\pi+\beta}^{\pi-\beta} E_{\phi 1} d\phi + \frac{\partial D_2}{\partial \phi_1} \int_{-\pi+\beta}^{\pi-\beta} E_{\phi 2} d\phi .$$

But the two interior sources are close together so that on the sphere,  $E_{\phi 1} \simeq E_{\phi 2}$  and therefore,

$$(23) \quad D_1(\phi_1) = K D_2(\phi_1)$$

where  $K$  is some constant. Hence it is required that (approximately)

$$(24) \quad 0 = \frac{\partial D_1}{\partial \phi_1} = \frac{\partial D_2}{\partial \phi_1} .$$

But this implies a stationary value of the current  $I$ , and therefore of the admittance,  $Y$ . Hence it is sufficient to use the simple condition

$$(25) \quad \frac{\partial Y}{\partial \phi_1} = 0$$

in order to optimize  $\phi_1$ .

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